SRVis: Towards Better Spatial Integration in Ranking Visualization

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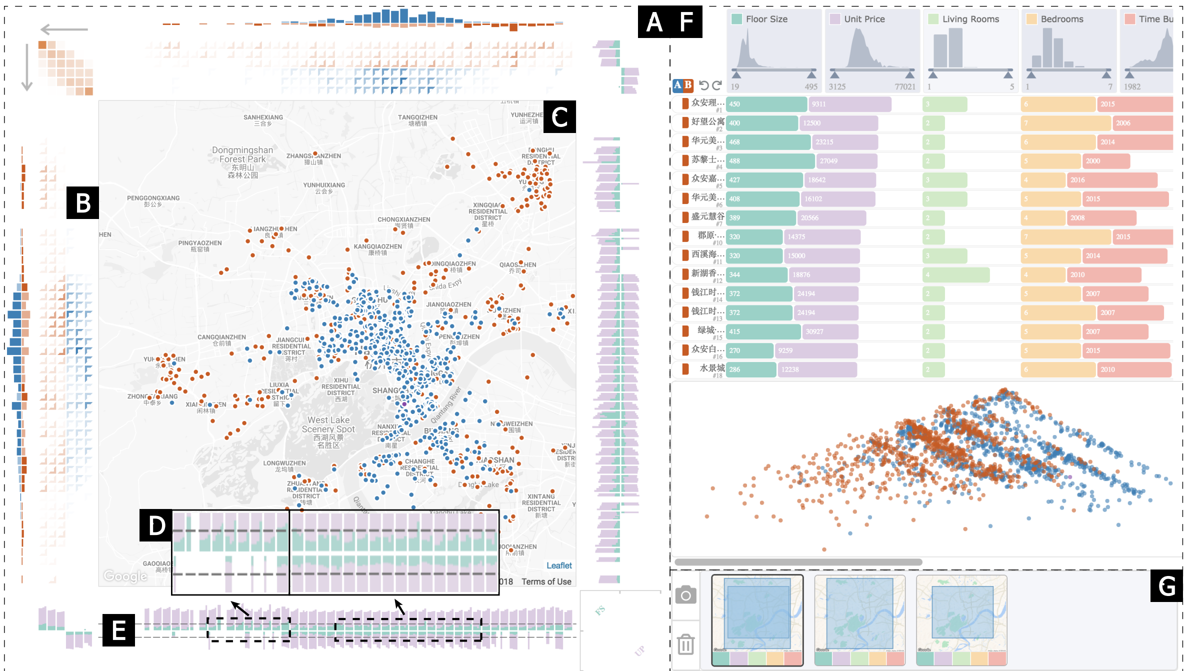

Fig. 1. The interface of SRVis. (A-E) The ranking view presents the rankings of alternatives and the cause of rankings with a matrix-based context-integrated visualization. Flexible spatial filtering features provided by B, C, and E enable users to conveniently explore and identify spatial patterns in the ranking datasets. (F) The inspector view adopts a table-based ranking technique to show all alternatives in the filtered region. A projection view is also utilized to assist users in finding similar alternatives based on their criteria. (G) The snapshot view allows users to save snapshots of rankings and criterion weights, such that users can compare these snapshots to find insights from the comparative analysis.

Abstract—Interactive ranking techniques have substantially promoted analysts' ability in making judicious and informed decisions effectively based on multiple criteria. However, the existing techniques cannot satisfactorily support the analysis tasks involved in ranking large-scale spatial alternatives, such as selecting optimal locations for chain stores, where the complex spatial contexts involved are essential to the decision-making process. Limitations observed in the prior attempts of integrating rankings with spatial contexts motivate us to develop a context-integrated visual ranking technique. Based on a set of generic design requirements we summarized by collaborating with domain experts, we propose SRVis, a novel spatial ranking visualization technique that supports efficient spatial multi-criteria decision-making processes by addressing three major challenges in the aforementioned context integration, namely, a) the presentation of spatial rankings and contexts, b) the scalability of rankings' visual representations, and c) the analysis of context-integrated spatial rankings. Specifically, we encode massive rankings and their cause with scalable matrix-based visualizations and stacked bar charts based on a novel two-phase optimization framework that minimizes the information loss, and the flexible spatial filtering and intuitive comparative analysis are adopted to enable the in-depth evaluation of the rankings and assist users in selecting the best spatial alternative. The effectiveness of the proposed technique has been evaluated and demonstrated with an empirical study of optimization methods, two case studies, and expert interviews.

Index Terms—Spatial ranking, visualization.

1 Introduction

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Visual ranking is an ubiquitous technique that helps users transparently make informed decisions via the intuitive visual representations of alternatives, such as spreadsheets, line charts, and stacked bar charts. This technique has been extensively adopted recently in various *multi-criteria decision-making* (MCDM) scenarios, including information retrieval [25], sports data analysis [15], and urban planning [30]. By sorting available alternatives in descending order based on the assigned scores, the preferences of decision-makers can be efficiently captured and characterized with visualizations.

Traditional visual ranking techniques excel at depicting the rankings of self-contained alternatives, which can be described with a set of criteria without introducing any additional context. However, challenges arise when applying these techniques to ranking spatial alternatives, which can be difficult to characterize with a few numerical criteria owing to the complexity of the corresponding spatial contexts. For example, in a classic spatial MCDM scenario in which users intend to search for an ideal house to rent, presenting rankings without contexts, such as road maps, may restrict users to a laborious browsing of a tedious list of candidates. Even with geo-filtering techniques, such as nearest neighbor queries [48], users cannot immediately grasp the detailed environments surrounding candidates from vague textual descriptions, such as addresses or distances.

To improve the reliability of spatial MCDM processes with comprehensive spatial contexts, spatial applications have implemented various context-integrated visual ranking techniques, which can be summarized into three categories, namely, point-, line-, and region-based techniques [22, 30, 50, 59] as introduced in Sect. 3. Most of these techniques have achieved preliminary integration either by directly encoding the rankings in the contexts or by illustrating the correspondence of alternatives between a ranked view and contexts. However, both approaches share three major drawbacks: a) the insufficient integration of contexts hinders the discovery of clear spatial patterns on the rankings and cause behind the rankings, which are crucial for making spatial decisions based on multiple criteria; b) these techniques do not scale well with the number of alternatives, producing severe visual clutters as the visual representations of these alternatives overlap with one another and occlude the underlying contexts; and c) the lack of flexible spatial filtering and comparison features in these techniques prohibits an in-depth evaluation of the rankings.

These limitations motivate us to propose a context-integrated visual ranking technique that facilitates the analysis of massive spatial rankings and supports effective spatial MCDM processes. However, developing such a technique poses three major challenges:

Presentation of spatial rankings and contexts. Seamlessly incorporating the rankings with the contexts significantly improves the efficiency of making reliable spatial decisions in various spatial MCDM scenarios. For example, establishing chain stores requires analysts to evaluate top-ranked locations against various spatial contexts, such as population density and nearby competitors. Such contextual evaluation highly demands a thoughtful design that tightly connects the numerically abstracted rankings and their cause with complex spatial contexts, which remain an unresolved challenge.

Scalability of rankings' visual representations. Spatial decision-making processes generally involve a substantial number of alternatives. For example, in a location selection scenario for billboards, users must evaluate numerous candidates because billboards can be placed nearly everywhere along roads. However, spaces left to visualize the rankings and their cause are extremely limited in the spatial contexts owing to the dense information encoded on the maps. Such limitation poses challenges on developing a clutter-free visual representation for massive alternatives, enabling the multi-level exploration of contextual rankings.

Analysis of context-integrated spatial rankings. Analytical features, such as filtering and comparison, help users locate and evaluate eminent alternatives and deliver reliable spatial decision-making processes. For example, iteratively constructing an effective ranking model may require users to inspect the top-ranked alternatives in a specific area and investigate the spatial variation in rankings as the criterion weights are changed. Such requirements demand an interactive implementation of flexible spatial filtering and extensive comparative analysis adapted to the contexts, which constitutes the third challenge.

This study is conducted in close collaboration with domain experts to summarize the design requirements for an effective spatial ranking visualization technique. Based on these requirements, we propose *SRVis*, a novel visual ranking technique that addresses the aforementioned challenges in the following aspects: a) *Presentation*: Rankings are tightly coupled with spatial contexts via matrices and stacked bars, which provide intuitive visual summaries of spatial ranking statistics vertically and horizontally; b) *Scalability*: The aggregated representation of rankings enables the effective visualization of millions of alternatives. Specifically, we encode the cause of rankings with compact stacked bars via a two-phase optimization framework, which minimizes the information loss and improves the bar layout based on greedy heuristics; c) *Analysis*: Flexible spatial filtering allows users to interactively locate interesting regions, which can be compared with different criterion weights to eventually enhance the fidelity of the final spatial decisions. Our main contributions are summarized as follows:

- We characterize the design requirements for an effective context-integrated visual ranking technique that supports generic spatial MCDM scenarios based on large-scale spatial rankings.
- We develop SRVis, a novel visual ranking technique that incorporates the interactive and scalable matrix-based representations of massive rankings with maps to enable the flexible filtering and comparative analysis of the rankings. In particular, we encode the cause of rankings with stacked bar charts based on a novel two-phase optimization framework.

2 Related Work

This section presents relevant studies on spatial and ranking visualization techniques.

2.1 Spatial Visualization

Generally, spatial visualization techniques can be categorized into three types based on which visual elements are used, namely, point-, line-, and region-based techniques [60]. Point-based techniques enable users to observe spatial objects or events by directly plotting their locations on spatial contexts, including maps, as points [5, 26, 31, 38]. The overlaps among points can be resolved using data aggregation techniques, such as KDE-based heatmaps [30, 32, 34]. Line-based techniques are frequently adopted in visualizing linear spatial data, such as trajectories with lines and curves [1,20,43]. Region-based techniques aggregate and present spatial datasets based on a predetermined space division [2]. This study leverages the aforementioned point-based techniques to visualize alternatives on spatial contexts.

2.2 Generic Ranking Visualization

Visual ranking emerges from the longstanding popularity of ranking-based decision making [7, 8]. By employing generic ranking visualizations, visual analytics systems can assist users in transparently ranking alternatives and making reliable and informed decisions. Inspired by the classification of visual ranking techniques proposed by Gratzl et al. [19], we further categorize these techniques based on the number of criteria involved in the rankings (univariate or multivariate) and the number of rankings per alternative computed from these criteria (single or multiple), thereby producing four types of visual ranking techniques: univariate single-ranking, univariate multiple-ranking, multivariate single-ranking, and multivariate multiple-ranking techniques.

Univariate single-ranking techniques are ubiquitous in daily life. These techniques generally order alternatives by a criterion and assign a ranking for each alternative. Among these techniques, spreadsheets [47] and bar charts [37] are the most extensively adopted ones. *Univariate multiple-ranking techniques* focus on evaluating and comparing multiple sets of rankings involving only one criterion, such as the rankings evolving with time [58]. Kidwell et al. [24] studied incomplete and partially ranked data by projecting multiple rankings with multi-dimensional scaling. Behrisch et al. [9] visualized multiple sets of rankings with small multiples. RankExplorer [44] and TrajRank [36] illustrated time-varying rankings with stack graphs. In addition, charts [6, 42] and glyphs [33, 57] are also extensively utilized to present such rankings. *Multivariate single-ranking techniques* visualize rankings involving multiple criteria of alternatives. Most of the approaches are table-based, such as ValueChart [13], ValueChart Plus [7], and Podium [55]. In addition, techniques like dimensional reduction [25] and glyphs [15] are also proposed in the prior studies. *Multivariate multiple-ranking techniques* are developed to enhance the analytical ability of single-ranking ones, helping users perceive and compare multiple sets of rankings based on several relevant criteria. Gratzl et al. [19] illustrated the ranking difference via a slope chart, in which the corresponding items between two ranked lists are connected with lines. Vuillemot and Perin [54] proposed a technique that tracks a time-varying multi-criteria ranking table with line charts and interaction-based animations.

This study develops a multivariate multiple-ranking technique that visualizes the spatial rankings generated from multiple criteria and help users analyze massive spatial ranking datasets conveniently by enabling them to compare multiple sets of rankings.

2.3 Spatial Ranking Visualization

In this section, we review the existing techniques for visualizing spatial rankings that involves the integration of rankings with their spatial contexts and summarize these techniques into three categories based on the types of visual elements used:

Point-based. Numerous techniques have been proposed to encode abstract rankings in the spatial contexts based on points. Most of these techniques utilize glyph representations, such as markers [59], colored circles [29], utility signs [3], and decision clocks [28]. Notably, Andrienko et al. [4] proposed utility symbols, a novel visualization technique for multivariate spatial rankings that creatively embeds the ranking of each object into a map with bar or pie charts, enabling the efficient context-aware analysis of multi-criteria spatial rankings. However, limited screen space prevents these techniques from scaling well with the number of alternatives, cluttering the underlying spatial contexts and producing severe overlapping between glyphs as the size of spatial data grows. Moreover, these techniques do not support direct visual comparisons between two different sets of rankings.

Line-based. This set of techniques aims to depict spatial rankings with lines, generally by connecting the corresponding alternatives based on lines between two coordinated views: one illustrating the abstract rankings with a table or list and the other depicting the spatial contexts of alternatives. Such linking can be achieved either by explicitly drawing lines [30] or by highlighting the selected alternatives on edge [22]. However, these techniques also have scalability issues and rely heavily on user interactions, which interfere with users' direct perception of the correspondence between abstract rankings and spatial contexts.

Region-based. In certain spatial decision-making scenarios wherein the alternatives involve regions, the rankings of these regions can be directly illustrated in spatial contexts using colors [22, 50, 51]. Despite the intuitiveness of this method, the scalability of such an encoding is strongly constrained by the discriminability of colors [46]. Another popular integration approach is to transform maps based on the rankings, such as cartogram [17, 40]. However, the interpretability of the transformed spatial contexts remains controversial [23, 49].

Motivated by the issues found in the aforementioned state-of-the-art techniques and the challenges concluded in the introduction, we develop SRVis, a point-based technique that visualizes large-scale spatial rankings and integrates these rankings with their spatial contexts intuitively without severe visual clutters.

3 Background and Requirement Analysis

This section summarizes several crucial requirements to design an effective visualization technique that enables the efficient presentation and exploration of massive spatial ranking datasets.

In the past year, we collaborated closely with two senior researchers from the business intelligence department of a large corporation, both of whom have decades of experience in geospatial data analysis and location-based marketing. Our initial goal was to design a visual analytics system that assists our collaborators in determining optimal locations for new chain stores based on multiple criteria, including the rent, population, the number of competitors nearby, etc. This process is known as the classic MCDM scenarios [52, 53], for which the solution framework generally comprises three steps: 1) identify relevant criteria and alternatives; 2) characterize decision makers' preferences; and 3) determine a ranking for each alternative based on preferences.

Efforts have been devoted to enable efficient MCDM processes with the visualization of rankings, such as scatterplots [25], tables [19, 55], and glyphs [15]. However, a few of our discussions with the domain experts on requirements reveal additional insights. Apart from the rankings, making informed decisions in spatial MCDM scenarios (e.g., the aforementioned one) also requires spatial contexts (e.g., maps). These contexts are essential to the decision-making effectiveness but difficult to capture with automatic processing [22]. Through a literature review, we found that the existing ranking visualization techniques could not satisfactorily integrate spatial rankings with their contexts due to several issues including the scalability as summarized in Sect. 2.3.

The lack of the considerate integration between the rankings and the spatial contexts motivates us to extend and generalize our approach to support the effective spatial decision-making based on the rankings of alternatives and consequently benefit a wider set of spatial applications. By following the *nested model for visualization design and validation* [39], we iteratively characterize the problem and identify the design requirements by reviewing relevant literature and conducting four interview sessions with the experts. We conclude the iteration with the requirements summarized as follows.

R.1: Present spatial rankings. *Where are the alternatives involved in ranking? In which area do the alternatives tend to have higher rankings? Are the rankings affected by additional spatial contexts?* Thus, a clear visual summary is required to assist users in efficiently comprehending the spatial distribution of the alternatives and their rankings. Moreover, the proposed design should be capable of incorporating this summary with supportive spatial contexts that enable a transparent decision-making process, such as road map, traffic map, and population density map, without introducing severe occlusions and visual clutters.

R.2: Unfold the cause of spatial rankings. *How are the spatial rankings computed from the criteria? Which area is dominated by a criterion? Are two criteria spatially correlated?* Since the numerically determined rankings cannot constantly provide an accurate representation of the performance of the alternatives, each criterion should be visualized in conjunction with spatial contexts to elaborate the formulation of these rankings and reveal insightful patterns of spatial trends. In addition, these visualized criteria can enable the informed adjustment of criterion weights by suggesting the reason for potential ranking anomalies based on the information provided by spatial contexts.

R.3: Scale with the number of alternatives. *How are large-scale spatial ranking datasets investigated? How can numerous alternatives and their rankings be preceived spatially?* Although spatial datasets generally comprise numerous alternatives, the scalability of state-of-the-art spatial ranking techniques remains unsatisfactory. To support an effective decision-making process, the proposed design should scale well with the number of alternatives in the datasets, thereby providing high-level insights into the rankings of these alternatives to assist further investigations of low-level details.

R.4: Support flexible spatial filtering. *How can analysts focus on the alternatives in a particularly interesting area? How can a filtered spatial view of the alternatives be obtained similar to a given one?* As a fundamental data exploration method, filtering allows users to dissect and analyze complex spatial ranking datasets with ease. By incorporating spatial filtering features in the proposed design, users can concentrate on the region of their interest in and exclude unwanted or irrelevant alternatives. Furthermore, the design should support spatial filtering based on the inherent hierarchy of the alternatives, such as administrative divisions.

R.5: Enable comparative analysis of spatial rankings. *What is the difference between the rankings of alternatives in two areas? How do the rankings differ spatially if criterion weights are changed? Which alternative is the best?* To help users extensively evaluate the performance of alternatives, the proposed design should provide two comparative features that facilitate a multi-faceted spatial analysis: a) visualize the difference in the rankings and their causes between two given regions; and b) indicate the variation in rankings caused by the modification of criterion weights. The design should also implement basic comparative features, such as sorting the alternatives based on the rankings generated from one or more criteria. Such features are definitely necessary when determining the best option.

4 Context-Integrated Visual Ranking Technique

This section presents SRVis, a novel context-integrated visual ranking technique that facilitates the effective presentation and analysis of massive spatial rankings. To support the requirements summarized in Sect. 3, SRVis comprises three coordinated views, namely, ranking (Fig. 1A), inspector (Fig. 1F), and snapshot (Fig. 1G) views. In the ranking view, the locations of alternatives are plotted on a map, along with two ranking matrices summarizing the statistics of rankings placed to the top and the left of the map (R.1, R.3). To the right and bottom of the map are two criteria charts that help users efficiently grasp the criteria distribution of spatially corresponded alternatives and understand the cause of rankings (R.2, R.3). In addition, the map, matrices, and criteria charts are linked together, such that users may brush on them to interactively locate interesting regions (R.4). The alternatives in the selected regions will then be presented in the inspector view. The

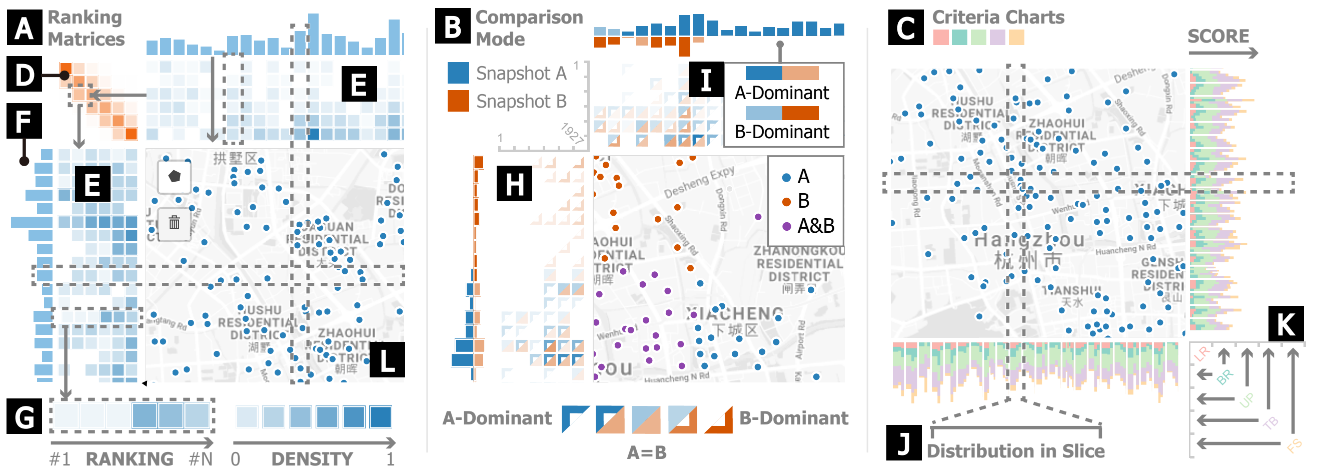


Fig. 2. The design of the ranking view. (A, D-G, L) A scalable matrix-based visual representation of spatial rankings based on the aggregation of the vertical and horizontal map slices; (B, H, I) Comparing two sets of rankings in the comparison mode of the ranking view; (C, J, K) The design of criteria charts that spatially depict the cause of rankings.

inspector view also provides detailed information on each alternative and allows users to filter and group alternatives based on the value and similarity of criteria (R.4). Furthermore, users can save regions with the weights of criteria in the snapshot view and perform multi-faceted comparative analyses between two recorded regions (R.5).

4.1 Ranking View

The ranking view (Fig. 1A) presents the rankings of massive alternatives and the cause of these rankings in conjunction with the spatial contexts illustrated with a map. The visual design of the ranking view is depicted in the following sections in three aspects, namely, the visualization of the spatial contexts, rankings, and the cause of rankings.

4.1.1 Visualizing Spatial Contexts

Spatial contexts help users efficiently grasp and evaluate the surrounding environments of alternatives in various spatial decision-making scenarios. Thus, SRVis adopts the map-centred exploratory approach [22] by visualizing the spatial contexts with a map (Fig. 2L), wherein small blue circles encode the locations of alternatives. We reduce the size of circles and use a consistent color for every circle to minimize the occlusion and visual interference of underlying spatial contexts. In addition, users can overlay additional spatial contexts on the map, such as the population density heatmap or the locations of competitors. They can also draw polygons on the map to create a selection of interesting alternatives, thereby enabling interactive spatial filtering (R.4).

4.1.2 Visualizing Rankings

To visualize the spatial distribution of massive rankings, we divide the rankings evenly into several *ranking groups* and encode the spatial distribution of these groups with a highly scalable visual representation (Fig. 2A) based on matrices and bar charts (R.1, R.3). The visual representation is further extended (Fig. 2B) to support the comparison between two different sets of rankings (R.5).

Encoding rankings with matrices. Two ranking matrices (Fig. 2E) are placed to the top and the left of the map, respectively. Each column of the top matrix and each row of the left matrix is aligned with a vertical or horizontal slice of the map. The alternatives covered by a map slice are partitioned based on their ranking groups and accumulated in the corresponding cells of the matrices. The cells (Fig. 2G) are arranged in descending order of ranking groups (i.e., from high- to low-ranked) from the outer to the inner of matrices, and the opacity of each cell encodes the density of alternatives in the corresponding ranking group. Moreover, a bar chart (Fig. 2F) is laid at the outer edge of each matrix to illustrate the number of alternatives covered by the corresponding map slices. Users can brush and select an eminent area on the matrices or bar charts and apply spatial and ranking filters simultaneously to focus on the alternatives belonging to the specified ranking groups in the designated region (R.4).

Encoding fluctuations in rankings. A fluctuation matrix (Fig. 2D) appears at the upper left corner of the ranking view when alternatives are re-ranked owing to the modification of criterion weights (R.5). The opacity of the cell at the i-th row and j-th column encodes:

$$f(i,j) = \frac{|R_i \rightarrow R_j|}{|R_i \cup R_j|},$$

where R_i and R_j correspond to the i-th and j-th ranking groups, respectively, and $R_i \rightarrow R_j$ represents the set of the alternatives in the ranking group R_j that previously belong to R_i. For intuitiveness, we add arrows at the edge of the matrix to signify the flow directions of the alternatives and allow users to interactively hover over a cell and highlight the corresponding ranking groups in the ranking matrices.

4.1.3 Visualizing Cause of Rankings

To better comprehend the formulation of rankings and identify spatial patterns in such formulation, users must be able to determine the cause of rankings spatially (R.2). Inspired by ValueCharts [13], we model the cause of rankings and present the spatial distribution of the cause with tailored stacked bar charts called criteria charts. In addition, given the scalability consideration (R.3), we optimize the bar layout in criteria charts with a novel two-phase framework based on greedy heuristics, enabling a scalable and legible design of the proposed charts.

Modeling the cause of rankings. This study ranks alternatives using the *simple additive weighting* (SAW) technique [21], which has been extensively adopted in various MCDM scenarios. Suppose m relevant criteria are identified, we denote the j-th normalized criterion value of the i-th alternative by c_{ij}. Then, scores are computed with

$$s_i = \sum_{j=1}^{m} w_j c_{ij},$$

where w_j is the weight assigned to the j-th criterion and the sum of weights $\sum_j w_j = 1$. Rankings can be further obtained from sorting alternatives by the computed scores in descending order. Hence, the weighted criteria that formulate the score of an alternative are particularly important for analysts to understand the cause of rankings. To facilitate the spatial interpretability of rankings, we integrate the visualization of weighted criteria with a map in the proposed design as described in the following section.

Encoding the spatial distribution of weighted criteria. Similar to the ranking matrices, two criteria charts (Fig. 2C) are placed to the right and the bottom of the map, respectively. Each chart consists of a series of criteria bars (Fig. 2J), associated with the alternatives covered by the corresponding vertical or horizontal map slices. In each criteria bar, the distribution of a weighted criterion is visualized with a colored compact bar chart. Moreover, each bar in these compact bar charts represents a group of alternatives, and the height of the bar encodes the mean of the weighted criterion values of these alternatives. To reflect the distribution of overall scores, these bar charts are further stacked on top of each other. Additionally, users can toggle the stacking baseline and change the stacking order of the criteria (Fig. 2K), thereby focusing on the distribution of a specific criterion or comparing between the distributions of multiple criteria. Furthermore, users can toggle the chart to show only the criteria values of the best- or the worst-performing alternatives. Thus, the design enables users to quickly grasp the spatial distribution of each criterion and the score and identify spatial patterns in the rankings and the cause of the alternatives' rankings.

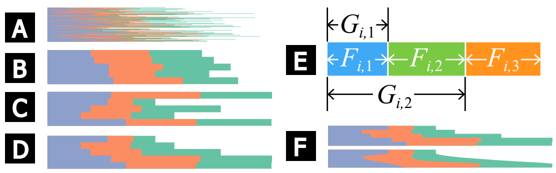


Fig. 3. (A) 100 compactly plotted stacked bars; (B) Sample alternatives by averaging without reordering; (C) The wiggling result obtained after phase I; (D) A legible layout obtained after phase II; (E) Compute G_{ij} from F_{ij}; (F) Owing to larger visual regions, step lines are more legible and easy to compare than curves at the borders of criteria bars.

Improving the layout of criteria bars. While the criteria are being visualized in the compact criteria bars, the distributions of the criteria may become illegible if alternatives are not aggregated. For example, Fig. 3A compactly plots the distributions of three criteria of a hundred randomly-generated alternatives with one stacked bar drawn for each alternative. However, almost no useful information can be obtained from such a plot because: a) the horizontal borders of each bar are indistinguishable due to excessive compression; and b) the bars of the same color (i.e., representing the same criterion) are not vertically continuous, leading to severe visual clutters. These two issues can be addressed by aggregating alternatives (scalability constraint) and reducing criterion wiggles (legibility constraint), respectively. However, as we shall see immediately, these two constraints inherently conflict with each other, thereby demanding a two-phase optimization framework.

Phase I: Improving the scalability via aggregation. To develop a scalable visual representation of criterion distributions, we sample the aforementioned stacked bars by taking the mean of the weighted criterion values of several continuous alternatives. Formally, we define p as the number of alternatives in each sample. p can be selected adaptively, such that the number of samples in a criteria bar will not exceed a given threshold, ensuring the visibility of the individual group of alternatives. Without the loss of generality, we assume the number of alternatives n is divisible by p. First, we denote the j-th weighted criterion of the i-th alternative with f_{ij}, such that $f_{ij} = w_j c_{ij}$. Then, we obtain n/p samples by averaging the weighted criteria of alternatives, where the value of the j-th criterion of the i-th sample is defined as

$$F_{ij} = \frac{1}{p} \sum_{k=p(i-1)+1}^{pi} f_{kj}.$$

Although a clearer layout of criteria bars has been established with grouped alternatives (Fig. 3B), such a layout cannot accurately reflect the real distribution of criterion values. In each sample, the local distributions of weighted criteria are lost during the averaging, resulting in a global distribution too smooth to observe important statistics. Therefore, the sampling order of alternatives must be carefully selected (i.e., reorder the alternatives optimally), such that the variance of weighted criterion values in each sample are minimized. However, this objective inherently conflicts with that of the legibility constraint, which inclines towards minimizing inter-sample variance to produce a smooth distribution. Hence, in this phase, we concentrate on the accuracy of sampled distributions, which helps users obtain correct information, and leave the optimization of legibility for the next phase after the alternatives are sampled.

To preserve an accurate criteria distribution after sampling by minimizing the intra-sample variance, we consider two situations, namely,

Algorithm 1 Minimize the `loss` based on greedy heuristics.

INPUT: the number of alternatives n, the number of criteria m, an array f of size $n \times m$ describing the weighted criteria of alternatives, and the sample size p.
OUTPUT: An optimized sequence of alternatives f'.

1: **procedure** LOCALLOSS(p, f, k) ▷ Compute the `loss` of the k-th sample of size p.
2: **return** $\frac{1}{m}\sum_{j=1}^{m}\sqrt{(\sum_{i=pk}^{p(k+1)-1}(f_{ij}-\frac{1}{p}\sum_{t=pk}^{p(k+1)-1}f_{tj})^2)/p}$
3: **procedure** GREEDY(f, p)
4: iter $\leftarrow 0$
5: **do**
6: iter $\leftarrow$ iter $+1$, swapped $\leftarrow$ `FALSE`
7: **for** $u \leftarrow 1$ **to** n/p **do**
8: **for** $i \leftarrow pu$ **to** $p(u+1)-1$ **do**
9: **for** $v \leftarrow u+1$ **to** n/p **do**
10: **for** $j \leftarrow pv$ **to** $p(v+1)-1$ **do**
11: oldLoss $\leftarrow$LOCALLOSS(p, f, u)+LOCALLOSS(p, f, v)
12: swap f_i and f_j ▷ Make a tentative swap.
13: newLoss $\leftarrow$LOCALLOSS(p, f, u)+LOCALLOSS(p, f, v)
14: **if** newLoss $<$ oldLoss **then** swapped $\leftarrow$ `TRUE`
15: **else** swap f_i and f_j ▷ Revert to the old sequence.
16: **while** swapped **and** iter $<$ `MAX_ITER`
17: **return** f

univariate and multivariate rankings. If the rankings of alternatives only involve a criterion, we can sort alternatives by their criterion value in ascending order, thereby obtaining an optimal sequence of alternatives, where the variance of each sample is minimized. In contrast, however, it is difficult to obtain an optimal solution for multi-criteria scenarios with similar methods. To characterize the information loss in the sampling process, we first derive a metric based on root-mean-square deviation:

$$\texttt{loss} = \frac{p}{nm}\sum_{i=1}^{n/p}\sum_{j=1}^{m}\sqrt{\frac{\sum_{k=p(i-1)+1}^{pi}(f_{kj}-F_{ij})^2}{p}},$$

where we compute the normalized deviation for each weighted criterion of every alternative from the mean of the corresponding sample of the criterion. Minimizing the `loss` is identical to the NP-hard *Job-Shop Scheduling* problem [18], where each stacked bar can be seen as a job scheduled to execute on n/p machines. However, the computational cost of obtaining a decent solution with combinatorial optimization by searching the permutations of alternatives is too expensive to support real-time map interactions. Therefore, we approximate the optima by adopting a greedy heuristic approach `GREEDY`, as illustrated in Alg. 1, where we iteratively swapping alternatives (cf. line 12) whenever it is possible to reduce the `loss` (cf. line 14). We also evaluate our approach against several other approaches including `RANDOM`, `SORTED`, and `BRUTE-FORCE` in Sect. 6.1. Eventually, alternatives are sampled and aggregated based on the generated order to maintain the accuracy of weighted criterion distributions.

Phase II: Improving the legibility via wiggle reduction. Sampled sequences obtained from the phase I can be seriously distorted because of the discontinuity of criterion stacked bars between samples (Fig. 3C). This can be alleviated by rearranging the order of the samples, minimizing the first derivatives of criterion values to create a smooth layout for criteria bars. Inspired by the design of streamgraphs [12], we denote the sum of $F_{i,1}, F_{i,2}, \ldots, F_{i,k}$ by $G_{i,k}$ (Fig. 3E) and adapt the *1-norm based weighted wiggles* proposed by Di Bartolomeo and Hu [16] as follows:

$$\texttt{wwiggles} = \sum_{i=1}^{n/p}\sum_{j=1}^{m-1} F_{i,j}\left(\frac{|G'_{i,j+1} - G'_{i,j}|}{2}\right),$$

where the derivative G'_{ij} can be computed with the central difference of adjacent samples, such that $G'_{ij} = (G_{i+1,j} - G_{i-1,j})/2$. The `wwiggles` tends to increase if the colored bars in the chart fluctuate, penalizing the wiggles between samples by the size of bars. Similarly, the approximation of optima can be obtained with the aforementioned swap-based greedy heuristic approach. With the `wwiggles` minimized, a smooth and legible stacked layout can be obtained for each criteria bar (Fig. 3D). Furthermore, we can apply such optimization to all criteria bars simul-

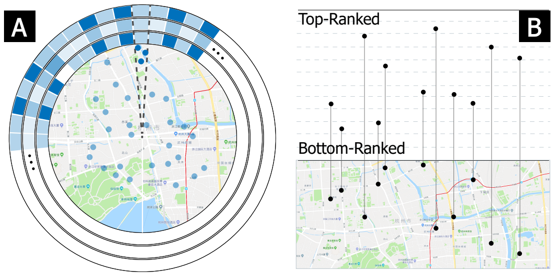


Fig. 4. Two design alternatives for the spatial ranking visualization. (A) Use circular maps instead of rectangular ones; (B) Illustrate rankings for each alternative with a vertically correspondent view;

taneously while restricting the swaps of alternatives to occurring inside the individual bars only, thereby enabling a refined global layout for the entire criteria chart.

4.1.4 Comparative Analysis of Rankings

Users must frequently compare different sets of rankings to identify the spatial patterns of alternatives (R.5). Generally, these ranking sets can be obtained in the following scenarios: a) users want to see the sensitivity of the rankings in a specific region with respect to a criterion by modifying its weight; and b) users want to locate the difference in the rankings of alternatives in two specific regions. In the scenarios where ranking sets cannot be obtained directly, such as the second one, we rank all alternatives first and then separate these rankings by regions, consequently creating two ranking sets.

To visualize the difference between two ranking sets, we divide each cell along its diagonal into blue and red triangles (Fig. 2H), and the opacity of each triangle encodes the density of alternatives belonging to the corresponding ranking set. A small white triangular arrow is also placed at the center of each cell, pointing towards the ranking set with high density of alternatives. Moreover, the bar charts (Fig. 2I) at the outer edge of the ranking matrices and criteria charts are replaced with diverging bar charts, each side of which illustrates the number of spatially corresponded alternatives in a ranking set. In the diverging bar charts, bars with higher opacity have more alternatives than their corresponding bars at the other side. Points on the map are rendered with three colors: a) blue: the point exists uniquely in the first ranking set; b) red: the point exists uniquely in the second ranking set; c) purple: the point exists in both ranking sets. Hence, the design enables users to conveniently perceive and compare the ranking sets on the matrices in terms of spatial and ranking distributions.

4.1.5 Design Alternatives

During the iterative development process of SRVis, we proposed several design alternatives that helped us derive and refine the final design. These alternatives are summarized in this section.

Visualizing context-integrated spatial rankings. The alternatives for ranking matrices are presented as follows.

Circular versus rectangular: Rankings can be integrated with either circular or rectangular maps. The design alternative illustrated in Fig. 4A adopts a radial approach, which encodes the rankings in the sectors of a disk around a circular map. Similarly, cells in the sectors are arranged in descending order of ranking groups from the outer to the inner part of the disk, and each sector is associated with the alternatives covered by the corresponding radial slice of the map. Although the radial design provides enhanced spatial correspondence between the rankings and the spatial contexts on the map, this design suffers the following three problems: a) users are less familiar with circular maps; b) circular maps waste corner space; c) the alternatives near the center of a circular map tend to suddenly jump to another nonadjacent map sector, causing confusion when users zoom or move the map. Therefore, a rectangular ranking visualization is implemented.

The scalability of rankings: Instead of aggregating rankings with ranking groups, we proposed another design alternative (Fig. 4B) that illustrates the direct correspondence between rankings and alternatives. The design comprises a map and a ranking view. Each alternative on the map vertically corresponds to and is linked with a dot in the ranking view, and the vertical positions of the dots encode the rankings of alternatives, thereby enabling an intuitive comparison of rankings. However, this design was rejected for three reasons: a) too many vertical lines produce severe visual clutters and occlude the spatial contexts; b) only the horizontal spatial distribution of rankings is reflected in the design; c) such a encoding is space inefficient, and thus, the discriminability of rankings suffers from visualizing thousands of alternatives with a single positional visual channel if the space is limited. Hence, we iterate the design by removing the visual elements that interfere with the spatial contexts, presenting the horizontal and vertical spatial distributions, and aggregating rankings with compact matrices. Such design leaves extra room for the spatial contexts while allowing users to compare detailed rankings in the inspector view.

Visualizing the cause of spatial rankings. ValueCharts [13] and LineUp [19] propose encoding the criteria involved in ranking with a stacked bar chart, in which each stacked bar corresponds to an alternative. However, such a design is unsuitable to embed with spatial contexts because alternatives are densely distributed on the map, which leaves little room to illustrate the cause of rankings that corresponds spatially with each alternative. Hence, we extend this approach and develop a compact design that is capable of visualizing a group of multi-criteria alternatives.

To encode the distributions of the criterion values and overall scores, we draw a stacked bar for each alternative that represents the score by stacking the weighted criterion values and compress these stacked bars with averaging sampling to improve legibility. One design alternative is to draw smooth curves similar to those in stream graphs instead of step lines by interpolating the edge of stacked bars (see Fig. 3F). However, we decide to stick with step lines because they tend to be more legible and easier to compare than curves in compact bars if the difference in the lengths of bars is significant.

4.2 Inspector View

To present the details (R.1) and cause (R.2) of rankings, we implement a tailored table-based ValueChart [13] in the inspector view (Fig. 1F), wherein each row corresponds to an alternative and each column corresponds to a criterion. The normalized criterion values of each alternative in a table row are illustrated with the bars scaled with the widths of columns, which are proportional to the weights of the corresponding criteria. Users can adjust the weight for each criterion, stack the values of multiple criteria, and rank alternatives based on the selected criteria. The history of weight adjustments has been recorded and can be controlled with undo and redo buttons to the left of columns. As such, the changes in the rankings of alternatives caused by weight modifications can be traced and replayed in the fluctuation matrix.

Furthermore, we project all alternatives with nonmetric multidimensional scaling [27] based on the Euclidean distances between criterion values onto a 2D projection view to demonstrate the potential similarity of alternatives. Users can identify clusters of alternatives with such projection and group alternatives accordingly in the inspector view to obtain a spatial overview of the rankings and the cause of rankings of similar alternatives (R.4). In addition, we visualize the criteria of grouped alternatives with their averages, since violent variations of criterion values in a cluster of similar alternatives do not occur frequently.

4.3 Snapshot View

The snapshot view (Fig. 1G) allows users to save a snapshot of the current spatial selection of alternatives and the weights of criteria to record interesting spatial patterns when the ranking datasets are explored. Each snapshot is visualized as a minimap of the area that covers the selected alternatives, with a stacked bar underneath illustrating the assignment of criterion weights. In addition, users can restore the snapshots to the ranking and inspector views by clicking on them.

4.4 Interactions

To reveal the spatial patterns of rankings, we implement flexible analytical features including spatial filtering (R.4) and comparison (R.5) with the interactions summarized as follows.

Multiple coordinated views. Alternatives are presented in both ranking and inspector views. To help users further investigate the detailed rankings in the inspector view based on the spatial patterns identified in the ranking view, we illustrate the correspondence of alternatives between these two views with interactions. When users hover over an alternative in either view, the size of the corresponding circle on the map will be increased, and the cells in ranking matrices, criteria bar, row in the inspector view, and projected point associated with the alternative will be highlighted. Hence, the effective search and analysis of alternatives are enabled across multiple coordinated views.

Spatially filtering alternatives. To empower intuitive and informed spatial filtering, SRVis allows users to create a spatial selection of alternatives flexibly. The selection can be constructed by brushing on the ranking matrices and criteria charts, allowing users to filter alternatives spatially based on the number of alternatives and the distribution of rankings and criteria in the corresponding map slices. In addition, users can draw polygons on the map to include or exclude the alternatives in the polygons from the selection. The selection can be further refined by removing alternatives in the inspector view, which maintains all alternatives in the selection.

Comparing snapshots. Users can make comparisons between snapshots by dragging a snapshot onto another in the snapshot view. While comparing two sets of rankings in the snapshots obtained from different regions, the ranking view will enter the comparison mode, in which each matrix cell is split in half (Fig. 2H). Then, each bar chart diverges to represent both ranking sets (Fig. 2I). Moreover, users can obtain the ranking difference (e.g., increased, decreased, unchanged, and new rankings) of a snapshot compared with another snapshot from the glyph generated for each row.

5 Implementation

We implement SRVis in JavaScript in conjunction with several libraries including Vue.js, D^3 [10], and Turf.js. SRVis specifically comprises two major parts, namely, frontend and backend. The backend driven by Node.js computes projections for the alternatives and serves the spatial ranking data for the frontend. The frontend running directly in web browsers presents the rankings with the techniques described above and assists users in analyzing the data through interactive visualizations.

During the iterative development process, two spatial ranking datasets are used to evaluate the effectiveness of SRVis: *House dataset* comprises 1,927 houses available for sale during December 2016 in Hangzhou, China. The criteria involved in the dataset include textual descriptions, coordinates, price, floor size, year built, and the number of living rooms and bedrooms; *Store dataset* describes 4,968 convenience stores in the same city. These descriptions comprises relevant criteria including the flow of potential customers, number of residents and competitors within two predefined radii, number of target customers labeled with two tags, and the infiltration ratio of Internet technologies.

As suggested by the domain experts, the benefit criteria (e.g. floor size and the number of bedrooms) are mapped linearly from $[\mathtt{min}, \mathtt{max}]$ to $[0,1]$, and the cost criteria (e.g. unit price) are mapped linearly from $[\mathtt{min}, \mathtt{max}]$ to $[0,1]$.

6 Evaluation

In this section, we thoroughly evaluate the proposed technique with an empirical study of loss optimization, two case studies, an expert interview, and a task-based user study.

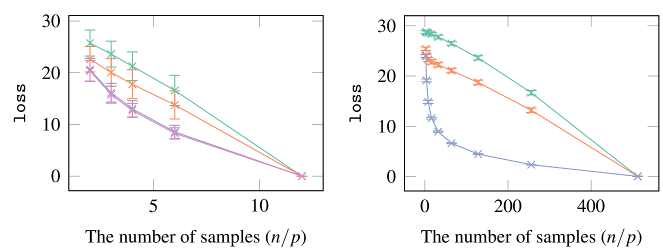
6.1 Loss Optimization

During the refinement of criteria bar layouts, we adopt a greedy heuristic approach `GREEDY` to minimize the information loss in the aggregation of alternatives. To evaluate the effectiveness of this approach, we compare it empirically with several other approaches, including `RANDOM`, `SORTED`, and `BRUTE-FORCE`. `RANDOM` simply shuffles alternatives randomly, resulting in the worst-case scenarios as the baseline of other approaches. `SORTED` sorts alternatives by the mean of their weighted criterion values in ascending order, which is capable of generating the optimal solutions for the alternatives involving only one criterion. `BRUTE-FORCE` generates the optimal order of alternatives that minimizes the `loss` by searching every possible permutation of alternatives with respect to samples, as illustrated in Alg. 2. Compared with the $O(pmn^2)$ time complexity of `GREEDY`, `BRUTE-FORCE` has an exponential complexity of $O(mn(\frac{n}{p})^n)$, which is apparently not suitable for larger datasets. Although studies [11] show that certain improvements can be made to such searching of permutations, the real-time processing of massive datasets to find optimal solutions is still out of grasp. As such, we generate small pseudo-datasets for the tests involving the comparison against the optimal solutions. Each dataset consists of 12 alternatives, and each alternative comprises 3 equally-weighted criteria, the values of which were randomly generated from a uniform real distribution between 0 and 100, inclusive.

Algorithm 2 Minimize the `loss` optimally with `BRUTE-FORCE`.

INPUT: the number of alternatives n, the number of criteria m, an array f of size $n \times m$ describing the weighted criteria of alternatives, and the sample size p.
OUTPUT: The optimal sequence of alternatives f'.

- *minLoss* $\leftarrow$ `Infinity`, $f' \leftarrow f$
- **procedure** GenPerm(f,p,u,c,f'') ▷ Searching a position for f_u in a new permutation f'', while c_i records the number of alternatives assigned to the i-th samples.
  - **if** $u = n+1$ **then** ▷ Determine if a new permutation has been obtained.
    - loss $\leftarrow \sum_{k=1}^{n/p}$ LocalLoss(p,f,k)
    - **if** loss $<$ minLoss **then** ▷ Determine if the current permutation is better.
      - minLoss $\leftarrow$ loss, $f' \leftarrow f''$
    - **return**
  - **for each** $c_i \in c$ **do**
    - **if** $c_i < p$ **then** ▷ Find a sample with an empty slot.
      - $c_i \leftarrow c_i + 1$, $f''_{c_i} \leftarrow f_u$
      - GenPerm(f,p,$u+1$,c,f'') ▷ Proceed to find a position for f_{u+1}.
- **procedure** BruteForce(f, p)
  - sampleSizes $\leftarrow \{0, \ldots\}$ of size n/p, $f'' \leftarrow \{$`nil`$, \ldots\}$ of size n
  - GenPerm(f,p,0,sampleSize,f'')
  - **return** f'



(a) Small (12 alternatives) (b) Large (512 alternatives)

Fig. 5. The information `loss` vs. the number of samples obtained on small (a) and large (b) pseudo-datasets with `RANDOM`, `SORTED`, `GREEDY`, and `BRUTE-FORCE` approaches.

We run `RANDOM`, `SORTED`, `GREEDY`, and `BRUTE-FORCE` against the sample size 2, 3, 4, and 6 for 500 times on randomized datasets and collect the average and standard deviation of the `loss` for each approach. The result is illustrated in Fig. 5(a). From the figure, we see that the `loss` decreases linearly w.r.t. the growth of the number of samples n/p in `RANDOM`, with the correlation coefficients [41] calculated to be -0.9990. In contrast, `GREEDY` generates solutions much closer to optimal ones obtained with `BRUTE-FORCE` than those with `SORTED`, which are 36.57% worse when $n/p = 4$, and `RANDOM`, which are 63.37% worse when $n/p = 4$ compared with `GREEDY`. The `loss` in `GREEDY` and `BRUTE-FORCE` both decreases logarithmically w.r.t. the growth of n/p, with the correlation coefficients calculated to be -0.9995 and -0.9997, respectively. Moreover, the `loss` of the solutions obtained with `GREEDY` is more stable than that of the solutions obtained with `SORTED` and `RANDOM`, compared with `BRUTE-FORCE`: $\frac{\sigma_{\mathtt{GREEDY}}}{\sigma_{\mathtt{BRUTE-FORCE}}} = 1.1635$, $\frac{\sigma_{\mathtt{SORTED}}}{\sigma_{\mathtt{BRUTE-FORCE}}} = 2.0676$, and $\frac{\sigma_{\mathtt{RANDOM}}}{\sigma_{\mathtt{BRUTE-FORCE}}} = 2.0528$ when $n/p = 4$.

This empirical evaluation shows that `GREEDY` performs significantly better and more stable than `SORTED` and the baseline `RANDOM`, and the trend in the `loss` of the solutions obtained with `GREEDY` is identical to that in the `loss` of the optimal solutions, from which we speculate the approximation ratio of `GREEDY` is bounded by some constant. To verify the reproducibility of the result, we run `RANDOM`, `SORTED`, and `GREEDY` against the sample size 2, 4, 8, 16, 32, 64, 128, and 256 for 500 times on large pseudo-datasets, which comprise 512 alternatives. Once again, the result on large datasets illustrated in the Fig. 5(b) demonstrates the significant improvement of `GREEDY` compared with other approaches and the logarithmic relation (correlation coefficient $r = -0.9870$) between the number of samples and the `loss` of approximated sequences.

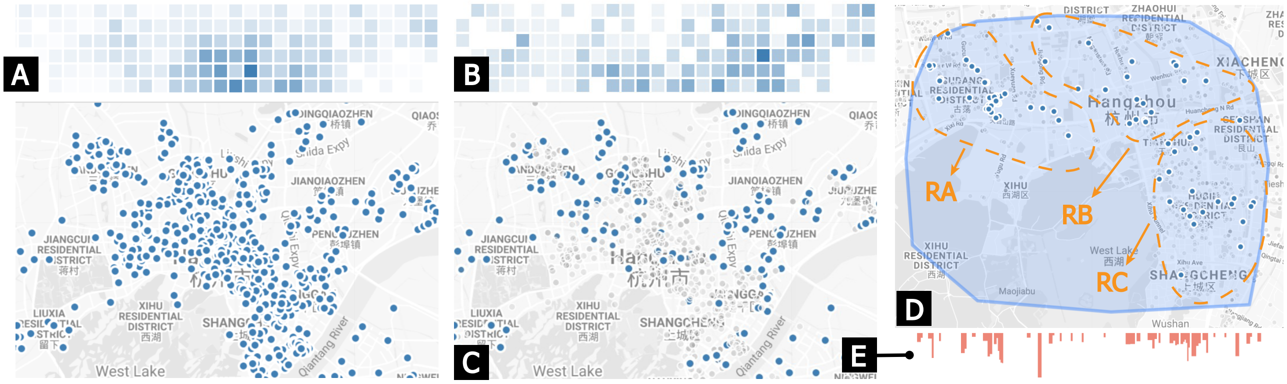


Fig. 6. (A) Low-ranked alternatives (50%-100%) were more concentrated in the center of the city than high-ranked ones (better than 50%); (B, C) After filtering out houses older than 5 years, which were mostly located in downtown area, the distribution in the ranking matrix became much more smooth; (D) Three clusters RA, RB, and RC with different distributions of alternatives can be identified on the map; (E) The customer flows (red) of RC did not show observable advantages over those of RA.

6.2 Case Studies

We conducted two case studies with two domain experts EA and EB, respectively, to evaluate the usability and effectiveness of SRVis. EA was specialized in house trading and interested in using SRVis to explore the houses available for sale and seek the opportunity to provide better house recommendations to his clients based on their requirements. EB had decades of experience in geospatial analysis and location-based marketing, and he would like to evaluate the performance of established convenience stores in the city with respect to several relevant criteria, such as population density and target customer coverage. Both of the experts had familiarized themselves with the proposed design and the structures of datasets.

6.2.1 Exploring the House Dataset

In this case study, we demonstrated the effectiveness of SRVis in helping experts obtain the spatial distribution of rankings, identify anomalies in the distribution, and find the cause of the anomalies based on experts' domain knowledge.

After EA loaded the house dataset into SRVis, he generated rankings for the alternatives in the dataset by selecting and grouping two equally-weighted criteria, namely, *floor size* (in sq. meters) and *unit price* (in dollars per sq. meter), which were considered by his clients the most important factors in finding the ideal house. Subsequently, the spatial distributions of the generated rankings were presented in the ranking matrices. EA immediately noticed that most of the low-ranked alternatives (bottom 50%) were in the center of the city, because in both of the horizontal (Fig. 6A) and vertical (Fig. 1B) ranking matrices, cells with dark colors in the lower parts of matrices were significantly more concentrated than those in the upper parts.

To figure out why the alternatives in the city center tended to receive lower rankings with floor size and unit price considered, EA brushed rectangular selections on the ranking matrices and divided all house alternatives into two sets by creating two snapshots from the corresponding selections, one comprising high-ranked (top 50%) alternatives and another one comprising remaining low-ranked (bottom 50%) alternatives. Then, he dragged a snapshot onto another to toggle the comparison mode of the ranking view (Fig. 1), thereby obtaining the difference in rankings (Fig. 1B) and the cause of rankings (Fig. 1E) between two sets of alternatives. In the criteria charts, EA modified the stacking order of criteria such that he could observe the individual distributions of floor size and unit price. He discovered that compared with low-ranked alternatives, most of the high-ranked alternatives were distributed around the city (Fig. 1C), and the floor sizes of high-ranked alternatives located in the suburb were larger than those of low-ranked ones located in downtown in the criteria charts (Fig. 1D). Therefore, EA suggested that these low-ranked alternatives might be old houses with small floor sizes, the geographical advantages of which also explained the reason why the unit price of these houses was slightly higher (bars representing the unit price were shorter because they were reversed to compute scores) than other newly-built houses in the same populated downtown area.

To confirm his hypothesis, EA left the comparison mode and adjusted the range filter of *time built* criterion such that only houses built in the recent five years were shown on the map. In the ranking view, he observed that most of the houses ruled out by the range filter, shown as gray circles on the map (Fig. 6C), were located in downtown, and the density distribution of alternatives illustrated in the matrices became much more smooth as he expected (Fig. 6B).

The above findings helped EA efficiently identify these old houses as cost-inefficient alternatives, which would not be recommended to those clients who were sensitive to the unit price. In addition, EA noted that these findings might also contribute to the iteration of the ranking model, where the surrounding environments of alternatives, such as the number of restaurants and shopping malls, would be included, such that the old downtown houses would gain higher rankings if his clients preferred the convenient accessibility to nearby facilities.

6.2.2 Exploring the Store Dataset

In this case study, we demonstrated the effectiveness of SRVis in interactively locating a small solution set of alternatives tailored for analysts' requirements via the combination of intuitive spatial distributions of criteria and flexible spatial filtering features.

The corporation EB worked for had been running an online logistic plaform for years, on which convenience store owners could register their stores and order supplies. To collaborate with some stores in upgrading their services and expanding corporation's business in local areas, EB would like to evaluate the performance of these stores and identify prominent ones from them. First, EB generated rankings with equal weights for all relevant criteria and focused on analyzing the promising alternatives in a populated district, which was located near a tourist attraction and comprised several major residential areas. With the spatial filtering tool, EB drew a polygon selection on the map to pick up all alternatives in this district (Fig. 6D). In addition, he also brushed and selected the first row of the horizontal ranking matrix to

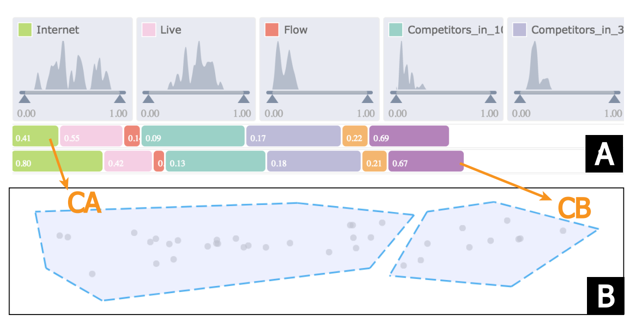


Fig. 7. (A) The number of nearby residents (Live) and customer flows (Flow) of CA were higher than those of CB, yet the alternatives in CB had significant advantages in the infiltration ratio of Internet technologies (Internet); (B) The projected alternatives formed a linear pattern.

obtain the locations of top-ranked ones among these alternatives.

In the ranking view, EB discovered that the top-ranked alternatives in the northwestern area *RA* of the district were more condensed than those in the northern and eastern areas *RB* and *RC*, as depicted on both of the map and counting bars. EB suggested that this could be because the densely-distributed office buildings and apartments in *RA* attracted many business owners to establish convenience stores. In contrast, in *RB* and *RC* resided a great number of large shopping malls, which might tend to obstruct the development of small stores. To confirm his hypothesis, EB evaluated these alternatives with the criteria charts, where he discovered that although the alternatives in *RB* and *RC* were located in downtown areas, the customer flows of these alternatives (Fig. 6E) did not exhibit observable advantages over those of the alternatives in *RA*, which were more close to residential areas. This could indicate that most of the customer flows in flourishing downtown areas were absorbed by large shopping malls, and the expensive rent in these areas further impeded the profitability of the alternatives in *RB* and *RC*. Therefore, EB decided to add another filter on the map to preserve the top-ranked alternatives in *RA* only.

In the inspector view, the multidimensional scaling projection of the alternatives caught EB's attention. The top-ranked alternatives in *RA*, which were projected into 2D points based on their similarities of criteria, formed a linear pattern in the view (Fig. 7B). EB wondered the reason behind such formulation, so he drew polygon selections in the projected view to create two clusters of alternatives *CA* and *CB*, one comprising the points on the left of the linear pattern and another comprising the points on the right, respectively. Interestingly, EB observed that two clusters exihibited different characteristics (Fig. 7A): most of the alternatives in *CA* were located near residential areas, thereby having more customer flows and target customers nearby; alternatives in *CB*, which were distributed more sparsely, showed clear advantages on the infiltration ratio of Internet technologies (green) over the alternatives in *CA*, indicating that store owners might be more open-minded to the collaborations yet to be proposed. Hence, based on the company strategy, EB decided to reach out for the stores in *CB* which he believed had better potential in profitability.

6.3 Expert Interview

We conducted interviews after the case studies to collect feedback on SRVis from the domain experts. Both EA and EB responded with a positive attitude towards the proposed technique, and they also spoke highly of the integrated design of ranking matrices. EA told us, "This novel matrix-based integration enables a third dimension for maps, where the efficient exploration of ranked houses is made interactive." EB confirmed the usability of our design: "I have never seen a geospatial analysis tool like this before, but it is quite intuitive and easy to learn." In addition, EB liked the design of criteria charts, from which he could "obtain the spatial distribution of one or several criteria from a higher level", but he also pointed out that it "becomes less useful if only a few alternatives remain on the map."

EA and EB offered some valuable advices as well. EB mentioned that in the comparison mode, diverging criteria bars could be confusing and difficult to be compared directly, because the values in a criteria bar were not ordered and aligned on both sides. EA agreed with EB's opinions and suggested that a dashed line could be added as a visual hint when users hover over the criteria charts to help users compare values between bars. Furthermore, he proposed a few modifications for better usability, including searching alternatives by their descriptions and visualizing criterion weights on snapshots. We have improved our system accordingly.

7 Discussion

In this section, we thoroughly discuss the advantages, limitations, and future work of the proposed technique SRVis.

Advantages. With the popularity of spatial analysis, SRVis is proposed to satisfy the huge demand of making informed spatial decisions based on the rankings of alternatives in numerous applications [22, 30, 56]. In such decision-making processes, the importance of presenting relevant spatial contexts has been recognized by the prior studies [22] in facilitating the effectiveness of decisions. Three challenges, including *presentation*, *scalability*, and *analysis*, are identified in formulating the tight integration of spatial contexts and rankings, as stated in the introduction. We show the advantages of our technique with respect to these challenges as follows: a) *Presentation*: The connections between the rankings, the cause of rankings and the alternatives on the map are established explicitly via the proposed matrix-based visualization without interfering spatial contexts. With the visualization serving as an overview, users can easily identify the spatial patterns of rankings and gain insights from interactive exploration, as demonstrated with the case studies shown above. b) *Scalability*: Compared with the existing approaches, SRVis can handle large-scale spatial ranking datasets much more smoothly due to the aggregation of rankings and the cause of rankings. By adopting flexible filtering features and multiple coordinated views, we compensate the information loss introduced by the aggregation of rankings and help users locate and analyze interesting regions and alternatives. Specifically, the scalability and legibility of the accurate aggregation of criteria distributions is maintained with a novel two-phase optimization framework. c) *Analysis*: SRVis enables users to filter and obtain the spatial distribution of rankings and the cause of rankings intuitively. In addition, users can compare the rankings between different regions and with different criterion weights. To the best of our knowledge, no prior technique has implemented such scalable and flexible analytical features for spatial rankings.

Limitations. Nonetheless, we also observe two limitations in the proposed technique. The first limitation exists in the greedy heuristic approach `GREEDY` we adopted to optimize the layout of criteria bars. Despite the good approximation achieved by this approach, the $O(pmn^2)$ time complexity of `GREEDY` remains too high to handle thousands of alternatives instantly. We argue that normally a map slice only covers a reasonable portion of alternatives, but there could be a faster method, which may also find a potentially better approximation with a formal theoretical bound. However, we believe it is beyond the scope of this study, which mainly focuses on the context integration of spatial rankings. The second limitation is that SRVis only considers linear ranking models, where rankings are obtained from the sum of weighted criteria. In contrast, non-linear ranking models, such as learning to rank [35], are also popular choices of ranking models. Nevertheless, we adopt linear ranking models for SRVis due to their wide applicability and intuitive interpretability, leaving the visualization of non-linear models for the future work. It is also worth noting that the multidimensional scaling projection in the inspector view might be unreliable in some cases. To amend this issue, the projection can be further improved by the designs and interactions proposed in the prior studies [14, 45].

Future work. We will continue working on SRVis to improve the performance of optimization algorithms and include the support for non-linear ranking models. In addition, visualizing streaming spatial rankings by integrating temporal rankings with spatial contexts will be an interesting direction to further enhance SRVis in the future.

8 Conclusion

This study proposes SRVis, a context-integrated technique for visualizing spatial rankings. In response to three identified challenges, namely, the presentation of spatial rankings and contexts, the scalability of rankings' visual representations, and the analysis of context-integrated spatial rankings, we collaborate closely with domain experts to characterize the generic requirements for the visualization of spatial rankings and design a novel matrix-based scalable visual representation for exploring and analyzing massive ranking datasets. In particular, we develop a two-phase optimization framework to integrate the cause of rankings with spatial contexts by considering both of the scalability and legibility of the proposed encodings. As a whole, SRVis enables the effective presentation and evaluation of large-scale spatial rankings with the tight and scalable integration of spatial contexts. The demo of our system is available at `http://zjuvis.org/srvis/`.

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